Proposition 6.2 (Class 12)

June 19, 2015

```
# parametrize the Hadamard square root of rank five


#[
# [  1,  0,  0,  0,  0,  1,  1,  1],
# [ y1,  0,  0,  0,  1,  1,  0,  0],
# [  0,  1,  0,  0,  0,  1,y12,y13],
# [  0, y4,  0,  0,y11,  1,  0,  0],
# [ y2,  0,  1,  0,  0,  0,  1,  0],
# [  0, y5, y7,  0,  0,  0,  1,  0],
# [ y3,  0,  0,  1,  0,  0,  0,  1],
# [  0, y6,  0, y9,  0,  0,  0,  1],
# [  0,  0, y8,y10,  1,  0,  0,  0]]

R.<y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12,y13>=QQ[];

M = matrix(R,[
 [  1,  0,  0,  0,  0,  1,  1,  1],
 [ y1,  0,  0,  0,  1,  1,  0,  0],
 [  0,  1,  0,  0,  0,  1,y12,y13],
 [  0, y4,  0,  0,y11,  1,  0,  0],
 [ y2,  0,  1,  0,  0,  0,  1,  0],
 [  0, y5, y7,  0,  0,  0,  1,  0],
 [ y3,  0,  0,  1,  0,  0,  0,  1],
 [  0, y6,  0, y9,  0,  0,  0,  1],
 [  0,  0, y8,y10,  1,  0,  0,  0]]); M

J=ideal(M.minors(6));
JJ=ideal(y1*y2*y3*y4*y5*y6*y7*y8*y9*y10*y11*y12*y13)
KK=J.saturation(JJ)
KK
[  1   0   0   0   0   1   1   1]
[ y1   0   0   0   1   1   0   0]
[  0   1   0   0   0   1 y12 y13]
[  0  y4   0   0 y11   1   0   0]
[ y2   0   1   0   0   0   1   0]
[  0  y5  y7   0   0   0   1   0]
[ y3   0   0   1   0   0   0   1]
[  0  y6   0  y9   0   0   0   1]
[  0   0  y8 y10   1   0   0   0]
```

```
(Ideal (y13 - 1, y12 - 1, y11 - 1, y10 - 1, y9 - 1, y8 - 1, y7 - 1, y4 + y5 + y6 - 1, y3 -
y6, y2 - y5, y1 + y5 + y6 - 1) of Multivariate Polynomial Ring in y1, y2, y3, y4, y5, y6,
y7, y8, y9, y10, y11, y12, y13 over Rational Field, 1)
```

```
# y13 - 1, y12 - 1, y11 - 1, y10 - 1, y9 - 1, y8 - 1, y7 - 1, y3 - y6, y2\
    - y5



R.<y1,y2,y3>=QQ[];

M = matrix(R,[
 [  1,  0,  0,  0,  0,  1,  1,  1],
 [ y1,  0,  0,  0,  1,  1,  0,  0],
 [  0,  1,  0,  0,  0,  1,  1,  1],
 [  0, y1,  0,  0,  1,  1,  0,  0],
 [ y2,  0,  1,  0,  0,  0,  1,  0],
 [  0, y2,  1,  0,  0,  0,  1,  0],
 [ y3,  0,  0,  1,  0,  0,  0,  1],
 [  0, y3,  0,  1,  0,  0,  0,  1],
 [  0,  0,  1,  1,  1,  0,  0,  0]]); M

J=ideal(M.minors(6));
JJ=ideal(y1*y2*y3)
KK=J.saturation(JJ)
KK
```

```
[ 1  0  0  0  0  1  1  1]
[y1  0  0  0  1  1  0  0]
[ 0  1  0  0  0  1  1  1]
[ 0 y1  0  0  1  1  0  0]
[y2  0  1  0  0  0  1  0]
[ 0 y2  1  0  0  0  1  0]
[y3  0  0  1  0  0  0  1]
[ 0 y3  0  1  0  0  0  1]
[ 0  0  1  1  1  0  0  0]
(Ideal (y1 + y2 + y3 - 1) of Multivariate Polynomial Ring in y1, y2, y3 over Rational
Field, 0)
```

```
# y1 + y2 + y3 - 1


# the Hadamard square root and the slack matrix can  be parametrized in \
   the following way


M = matrix(R,[
 [           1,           0,  0,  0,  0,  1,  1,  1],
 [          y1,           0,  0,  0,  1,  1,  0,  0],
 [           0,           1,  0,  0,  0,  1,  1,  1],
 [           0,          y1,  0,  0,  1,  1,  0,  0],
 [          y2,           0,  1,  0,  0,  0,  1,  0],
 [           0,          y2,  1,  0,  0,  0,  1,  0],
```

```
 [   (1-y1-y2),           0,  0,  1,  0,  0,  0,  1],
 [           0,   (1-y1-y2),  0,  1,  0,  0,  0,  1],
 [           0,           0,  1,  1,  1,  0,  0,  0]]);
```

```
# thus y1^2=x1, y2^2=x2, (1-y1-y2)^2=1-x1-x2

%macaulay2
R = QQ[y1,y2,x1,x2,MonomialOrder => Eliminate 2];
I = ideal(y1^2-x1,y2^2-x2,(1-y1-y2)^2-1+x1+x2);
G = selectInSubring(1,gens gb I);
newpoly = G_0_0;
toString factor newpoly
```

```
Ideal of R

       1      1
Matrix R  <--- R


(x1^4+2*x1^3*x2+3*x1^2*x2^2+2*x1*x2^3+x2^4-2*x1^3-2*x2^3+x1^2-2*x1*x2+x2^2)
```