

# Proposition 7.3 (Class 18)

June 19, 2015

```
# parametrize the Hadamard square root of rank five

#[
#[ 1, 1, 0, 0, 0, 0, 0, 0],
#[ 0, 0, 1, 0, 1, 0, 1, 0],
#[ 0, 0, 1, 0, y3, 0, 0, 1],
#[ 0, 1, y1, 0, 0, 1, y9, 0],
#[ 0, 1, 1, 0, 0, y6, 0, y12],
#[ 1, 0, 0, y2, y4, 0, y10, 0],
#[ 1, 0, 0, 1, y5, 0, 0, y13],
#[ 0, 0, 0, 1, 0, y7, y11, 0],
#[ 0, 0, 0, 1, 0, y8, 0, y14]]

R.<y1,y2,y3,y4,y5,y6,y7,y8,y9,y10,y11,y12,y13,y14>=QQ[];

M = matrix(R,[
[ 1, 1, 0, 0, 0, 0, 0, 0],
[ 0, 0, 1, 0, 1, 0, 1, 0],
[ 0, 0, 1, 0, y3, 0, 0, 1],
[ 0, 1, y1, 0, 0, 1, y9, 0],
[ 0, 1, 1, 0, 0, y6, 0, y12],
[ 1, 0, 0, y2, y4, 0, y10, 0],
[ 1, 0, 0, 1, y5, 0, 0, y13],
[ 0, 0, 0, 1, 0, y7, y11, 0],
[ 0, 0, 0, 1, 0, y8, 0, y14]]); M

J=ideal(M.minors(6));
JJ=ideal(y1*y2*y3*y4*y5*y6*y7*y8*y9*y10*y11*y12*y13*y14)
KK=J.saturation(JJ)
KK
[ 1 1 0 0 0 0 0 0]
[ 0 0 1 0 1 0 1 0]
[ 0 0 1 0 y3 0 0 1]
[ 0 1 y1 0 0 1 y9 0]
[ 0 1 1 0 0 y6 0 y12]
[ 1 0 0 y2 y4 0 y10 0]
[ 1 0 0 1 y5 0 0 y13]
[ 0 0 0 1 0 y7 y11 0]
[ 0 0 0 1 0 y8 0 y14]
(Ideal (y12 + y13 - y14 - 1, y11 - y14, y10 - y13, y9 + y13 - y14 - 1, y8 - 1, y7 - 1, y6
```

- 1,  $y_5 - 1$ ,  $y_4 - 1$ ,  $y_3 - 1$ ,  $y_2 - 1$ ,  $y_1 - 1$ ) of Multivariate Polynomial Ring in  $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}, y_{11}, y_{12}, y_{13}, y_{14}$  over Rational Field, 1)

```
# y11 - y14, y10 - y13, y8 - 1, y7 - 1, y6 - 1, y5 - 1, y4 - 1, y3 - 1, \
  y2 - 1, y1 - 1
```

```
# y9 + y13 - y14 - 1 => y14 = y9*y13 => y11 = y9*y10
```

```
# y9 + y13 - y14 - 1, y12 + y13 - y14 - 1 => y12 = y9
```

```
R.<y9,y10>=QQ[];
```

```
M = matrix(R,[
[ 1, 1, 0, 0, 0, 0, 0, 0],
[ 0, 0, 1, 0, 1, 0, 1, 0],
[ 0, 0, 1, 0, 1, 0, 0, 1],
[ 0, 1, 1, 0, 0, 1, y9, 0],
[ 0, 1, 1, 0, 0, 1, 0, y9],
[ 1, 0, 0, 1, 1, 0, y10, 0],
[ 1, 0, 0, 1, 1, 0, 0, y10],
[ 0, 0, 0, 1, 0, 1, y9*y10, 0],
[ 0, 0, 0, 1, 0, 1, 0, y9*y10]]); M
```

```
J=ideal(M.minors(6));
```

```
JJ=ideal(y9*y10)
```

```
KK=J.saturation(JJ)
```

```
KK
```

```
[ 1 1 0 0 0 0 0 0]
[ 0 0 1 0 1 0 1 0]
[ 0 0 1 0 1 0 0 1]
[ 0 1 1 0 0 1 y9 0]
[ 0 1 1 0 0 1 0 y9]
[ 1 0 0 1 1 0 y10 0]
[ 1 0 0 1 1 0 0 y10]
[ 0 0 0 1 0 1 y9*y10 0]
[ 0 0 0 1 0 1 0 y9*y10]
```

(Ideal ( $y_9*y_{10} - y_9 - y_{10} + 1$ ) of Multivariate Polynomial Ring in  $y_9, y_{10}$  over Rational Field, 0)

```
# y9*y10 - y9 - y10 + 1
```

```
# results
```

```
# for y9 = 1 the slack matrix has the form
```

```
M = matrix(R,[
[ 1, 1, 0, 0, 0, 0, 0, 0],
[ 0, 0, 1, 0, 1, 0, 1, 0],
[ 0, 0, 1, 0, 1, 0, 0, 1],
[ 0, 1, 1, 0, 0, 1, 1, 0],
[ 0, 1, 1, 0, 0, 1, 0, 1],
[ 1, 0, 0, 1, 1, 0, y10, 0],
[ 1, 0, 0, 1, 1, 0, 0, y10],
```

```
[ 0, 0, 0, 1, 0, 1,y10, 0],  
[ 0, 0, 0, 1, 0, 1, 0,y10]]);
```

```
# for y10=1 the slack matrix has the form
```

```
M = matrix(R,[  
[ 1, 1, 0, 0, 0, 0, 0, 0],  
[ 0, 0, 1, 0, 1, 0, 1, 0],  
[ 0, 0, 1, 0, 1, 0, 0, 1],  
[ 0, 1, 1, 0, 0, 1, y9, 0],  
[ 0, 1, 1, 0, 0, 1, 0, y9],  
[ 1, 0, 0, 1, 1, 0, 1, 0],  
[ 1, 0, 0, 1, 1, 0, 0, 1],  
[ 0, 0, 0, 1, 0, 1, y9, 0],  
[ 0, 0, 0, 1, 0, 1, 0, y9]]);
```

```
M.matrix_from_rows_and_columns([0,1,2,5,6,3,4,7,8],[0,1,4,5,2,3,6,7])
```

```
[ 1 1 0 0 0 0 0 0]  
[ 0 0 1 0 1 0 1 0]  
[ 0 0 1 0 1 0 0 1]  
[ 1 0 1 0 0 1 1 0]  
[ 1 0 1 0 0 1 0 1]  
[ 0 1 0 1 1 0 y9 0]  
[ 0 1 0 1 1 0 0 y9]  
[ 0 0 0 1 0 1 y9 0]  
[ 0 0 0 1 0 1 0 y9]
```