Proposition 7.3 (Class 20)

June 19, 2015

```
# parametrize the Hadamard square root of rank five



# 1)

#[
# [1,  0,  0,  0,  0,    1,   0],
# [1,  0,  0,  0,  0,    0,  y1],
# [0,  1,  0,  1,  0,    1,   0],
# [0,  1,  0,  1,  0,    0,   1],
# [0,  1,  0,  0,  1,    1,   0],
# [0,  1,  0,  0,  1,    0,   1],
# [0,  0,  1,  1,  0,   y2,   0],
# [0,  0,  1,  1,  0,    0,  y2],
# [0,  0,  1,  0,  1,   y2,   0],
# [0,  0,  1,  0,  1,    0,  y2]]


# 2)

#[
# [1,  0,  0,  0,  0,    1,   0],
# [1,  0,  0,  0,  0,    0,  y1],
# [0,  1,  0,  1,  0,    1,   0],
# [0,  1,  0, y2,  0,    0,   1],
# [0,  1,  0,  0,  1,    1,   0],
# [0,  1,  0,  0, y2,    0,   1],
# [0,  0,  1,  1,  0,    1,   0],
# [0,  0,  1, y2,  0,    0,   1],
# [0,  0,  1,  0,  1,    1,   0],
# [0,  0,  1,  0, y2,    0,   1]]


# 1)

R.<y1,y2>=QQ[];

M = matrix(R,[
 [1,  0,  0,  0,  0,    1,   0],
```

```
 [1,  0,  0,  0,  0,    0,  y1],
 [0,  1,  0,  1,  0,    1,   0],
 [0,  1,  0,  1,  0,    0,   1],
 [0,  1,  0,  0,  1,    1,   0],
 [0,  1,  0,  0,  1,    0,   1],
 [0,  0,  1,  1,  0,   y2,   0],
 [0,  0,  1,  1,  0,    0,  y2],
 [0,  0,  1,  0,  1,   y2,   0],
 [0,  0,  1,  0,  1,    0,  y2]]); M

J=ideal(M.minors(6));
JJ=ideal(y1*y2)
KK=J.saturation(JJ)
KK
```

```
[ 1  0  0  0  0  1  0]
[ 1  0  0  0  0  0 y1]
[ 0  1  0  1  0  1  0]
[ 0  1  0  1  0  0  1]
[ 0  1  0  0  1  1  0]
[ 0  1  0  0  1  0  1]
[ 0  0  1  1  0 y2  0]
[ 0  0  1  1  0  0 y2]
[ 0  0  1  0  1 y2  0]
[ 0  0  1  0  1  0 y2]
(Ideal (y1 - 1) of Multivariate Polynomial Ring in y1, y2 over Rational Field, 0)
```

```
# 1)

# results

M = matrix(R,[
 [1,  0,  0,  0,  0,    1,   0],
 [1,  0,  0,  0,  0,    0,   1],
 [0,  1,  0,  1,  0,    1,   0],
 [0,  1,  0,  1,  0,    0,   1],
 [0,  1,  0,  0,  1,    1,   0],
 [0,  1,  0,  0,  1,    0,   1],
 [0,  0,  1,  1,  0,   y2,   0],
 [0,  0,  1,  1,  0,    0,  y2],
 [0,  0,  1,  0,  1,   y2,   0],
 [0,  0,  1,  0,  1,    0,  y2]]); M
```

```
[ 1  0  0  0  0  1  0]
[ 1  0  0  0  0  0  1]
[ 0  1  0  1  0  1  0]
[ 0  1  0  1  0  0  1]
[ 0  1  0  0  1  1  0]
[ 0  1  0  0  1  0  1]
[ 0  0  1  1  0 y2  0]
[ 0  0  1  1  0  0 y2]
[ 0  0  1  0  1 y2  0]
[ 0  0  1  0  1  0 y2]
```

```
# 2)

R.<y1,y2>=QQ[];

M = matrix(R,[
 [1,  0,  0,  0,  0,    1,   0],
 [1,  0,  0,  0,  0,    0,  y1],
 [0,  1,  0,  1,  0,    1,   0],
 [0,  1,  0, y2,  0,    0,   1],
 [0,  1,  0,  0,  1,    1,   0],
 [0,  1,  0,  0, y2,    0,   1],
 [0,  0,  1,  1,  0,    1,   0],
 [0,  0,  1, y2,  0,    0,   1],
 [0,  0,  1,  0,  1,    1,   0],
 [0,  0,  1,  0, y2,    0,   1]]); M

J=ideal(M.minors(6));
JJ=ideal(y1*y2)
KK=J.saturation(JJ)
KK
[ 1  0  0  0  0  1  0]
[ 1  0  0  0  0  0 y1]
[ 0  1  0  1  0  1  0]
[ 0  1  0 y2  0  0  1]
[ 0  1  0  0  1  1  0]
[ 0  1  0  0 y2  0  1]
[ 0  0  1  1  0  1  0]
[ 0  0  1 y2  0  0  1]
[ 0  0  1  0  1  1  0]
[ 0  0  1  0 y2  0  1]
(Ideal (y2 - 1, y1 - 1) of Multivariate Polynomial Ring in y1, y2 over Rational Field, 0)

# thus the results in the case 2) are included in the case 1)
```