Proposition 7.4 (Class 28)

June 19, 2015

```
# parametrize the Hadamard square root of rank five

#1)

#[
# [0, 1, 0, 1, 0, 1, 0],
# [0, 1, 0, 1, 0, 0, 1],
# [0, 1, 0, 0, 1,y1, 0],
# [0, 1, 0, 0, 1, 0,y1],
# [0, 0, 1, 1, 0, 1, 0],
# [0, 0, 1, 1, 0, 0, 1],
# [0, 0, 1, 0, 1,y1, 0],
# [0, 0, 1, 0, 1, 0,y1],
# [1, 0, 0, 1, 0,y2, 0],
# [1, 0, 0,y3, 0, 0,y4],
# [1, 0, 0, 0,y5,y6, 0],
# [1, 0, 0, 0,y7, 0,y8]]

#2)

#[
# [0, 1, 0, 1, 0, 1, 0],
# [0, 1, 0, 1, 0, 0, 1],
# [0, 1, 0, 0, 1, 1, 0],
# [0, 1, 0, 0, 1, 0, 1],
# [0, 0, 1, 1, 0,y1, 0],
# [0, 0, 1, 1, 0, 0,y1],
# [0, 0, 1, 0, 1,y1, 0],
# [0, 0, 1, 0, 1, 0,y1],
# [1, 0, 0, 1, 0,y2, 0],
# [1, 0, 0,y3, 0, 0,y4],
# [1, 0, 0, 0,y5,y6, 0],
# [1, 0, 0, 0,y7, 0,y8]]


# 1)

R.<y1,y2,y3,y4,y5,y6,y7,y8>=QQ[];

M = matrix(R,[
```

```
[0, 1, 0, 1, 0, 1, 0],
[0, 1, 0, 1, 0, 0, 1],
[0, 1, 0, 0, 1,y1, 0],
[0, 1, 0, 0, 1, 0,y1],
[0, 0, 1, 1, 0, 1, 0],
[0, 0, 1, 1, 0, 0, 1],
[0, 0, 1, 0, 1,y1, 0],
[0, 0, 1, 0, 1, 0,y1],
[1, 0, 0, 1, 0,y2, 0],
[1, 0, 0,y3, 0, 0,y4],
[1, 0, 0, 0,y5,y6, 0],
[1, 0, 0, 0,y7, 0,y8]]); M
J=ideal(M.minors(6));
JJ=ideal(y1*y2*y3*y4*y5*y6*y7*y8)
KK=J.saturation(JJ)
KK
```

```
[ 0  1  0  1  0  1  0]
[ 0  1  0  1  0  0  1]
[ 0  1  0  0  1 y1  0]
[ 0  1  0  0  1  0 y1]
[ 0  0  1  1  0  1  0]
[ 0  0  1  1  0  0  1]
[ 0  0  1  0  1 y1  0]
[ 0  0  1  0  1  0 y1]
[ 1  0  0  1  0 y2  0]
[ 1  0  0 y3  0  0 y4]
[ 1  0  0  0 y5 y6  0]
[ 1  0  0  0 y7  0 y8]
(Ideal (y7 - 1, y6 - y8, y5 - 1, y3 - 1, y2 - y4, y1 + y4 - y8 - 1) of Multivariate
Polynomial Ring in y1, y2, y3, y4, y5, y6, y7, y8 over Rational Field, 0)
```

```
# 1)

# y7 - 1, y6 - y8, y5 - 1, y3 - 1, y2 - y4
#  y1 + y4 - y8 - 1 => y8 = y1*y4 = y1*y2

R.<y1,y2>=QQ[];

M = matrix(R,[
[0, 1, 0, 1, 0,    1,    0],
[0, 1, 0, 1, 0,    0,    1],
[0, 1, 0, 0, 1,   y1,    0],
[0, 1, 0, 0, 1,    0,   y1],
[0, 0, 1, 1, 0,    1,    0],
[0, 0, 1, 1, 0,    0,    1],
[0, 0, 1, 0, 1,   y1,    0],
[0, 0, 1, 0, 1,    0,   y1],
[1, 0, 0, 1, 0,   y2,    0],
[1, 0, 0, 1, 0,    0,   y2],
[1, 0, 0, 0, 1,y1*y2,    0],
[1, 0, 0, 0, 1,    0,y1*y2]]); M
```

```
J=ideal(M.minors(6));
JJ=ideal(y1*y2)
KK=J.saturation(JJ)
KK
```

```
[    0     1     0     1     0     1     0]
[    0     1     0     1     0     0     1]
[    0     1     0     0     1    y1     0]
[    0     1     0     0     1     0    y1]
[    0     0     1     1     0     1     0]
[    0     0     1     1     0     0     1]
[    0     0     1     0     1    y1     0]
[    0     0     1     0     1     0    y1]
[    1     0     0     1     0    y2     0]
[    1     0     0     1     0     0    y2]
[    1     0     0     0     1 y1*y2     0]
[    1     0     0     0     1     0 y1*y2]
(Ideal (y1*y2 - y1 - y2 + 1) of Multivariate Polynomial Ring in y1, y2 over Rational
Field, 0)
```

```
# 1)

#  y1*y2 - y1 - y2 + 1

# results

# for y1 = 1 the slack matrix has the form

M = matrix(R,[
[0, 1, 0, 1, 0,    1,    0],
[0, 1, 0, 1, 0,    0,    1],
[0, 1, 0, 0, 1,    1,    0],
[0, 1, 0, 0, 1,    0,    1],
[0, 0, 1, 1, 0,    1,    0],
[0, 0, 1, 1, 0,    0,    1],
[0, 0, 1, 0, 1,    1,    0],
[0, 0, 1, 0, 1,    0,    1],
[1, 0, 0, 1, 0,   y2,    0],
[1, 0, 0, 1, 0,    0,   y2],
[1, 0, 0, 0, 1,   y2,    0],
[1, 0, 0, 0, 1,    0,   y2]]);

# for y2 = 1 the slack matrix has the form

M = matrix(R,[
[0, 1, 0, 1, 0,    1,    0],
[0, 1, 0, 1, 0,    0,    1],
[0, 1, 0, 0, 1,   y1,    0],
[0, 1, 0, 0, 1,    0,   y1],
[0, 0, 1, 1, 0,    1,    0],
[0, 0, 1, 1, 0,    0,    1],
[0, 0, 1, 0, 1,   y1,    0],
```

```
[0, 0, 1, 0, 1,    0,   y1],
[1, 0, 0, 1, 0,    1,    0],
[1, 0, 0, 1, 0,    0,    1],
[1, 0, 0, 0, 1,   y1,    0],
[1, 0, 0, 0, 1,    0,   y1]]);
```

```
# 2)

R.<y1,y2,y3,y4,y5,y6,y7,y8>=QQ[];

M = matrix(R,[
[0, 1, 0, 1, 0, 1, 0],
[0, 1, 0, 1, 0, 0, 1],
[0, 1, 0, 0, 1, 1, 0],
[0, 1, 0, 0, 1, 0, 1],
[0, 0, 1, 1, 0,y1, 0],
[0, 0, 1, 1, 0, 0,y1],
[0, 0, 1, 0, 1,y1, 0],
[0, 0, 1, 0, 1, 0,y1],
[1, 0, 0, 1, 0,y2, 0],
[1, 0, 0,y3, 0, 0,y4],
[1, 0, 0, 0,y5,y6, 0],
[1, 0, 0, 0,y7, 0,y8]]); M
J=ideal(M.minors(6));
JJ=ideal(y1*y2*y3*y4*y5*y6*y7*y8)
KK=J.saturation(JJ)
KK
```

```
[ 0  1  0  1  0  1  0]
[ 0  1  0  1  0  0  1]
[ 0  1  0  0  1  1  0]
[ 0  1  0  0  1  0  1]
[ 0  0  1  1  0 y1  0]
[ 0  0  1  1  0  0 y1]
[ 0  0  1  0  1 y1  0]
[ 0  0  1  0  1  0 y1]
[ 1  0  0  1  0 y2  0]
[ 1  0  0 y3  0  0 y4]
[ 1  0  0  0 y5 y6  0]
[ 1  0  0  0 y7  0 y8]
(Ideal (y7 - 1, y6 - y8, y5 - 1, y4 - y8, y3 - 1, y2 - y8) of Multivariate Polynomial Ring
in y1, y2, y3, y4, y5, y6, y7, y8 over Rational Field, 0)
```

```
# 2)

#  y7 - 1, y6 - y8, y5 - 1, y4 - y8, y3 - 1, y2 - y8

# results

M = matrix(R,[
[0, 1, 0, 1, 0, 1, 0],
[0, 1, 0, 1, 0, 0, 1],
```

```
[0, 1, 0, 0, 1, 1, 0],
[0, 1, 0, 0, 1, 0, 1],
[0, 0, 1, 1, 0,y1, 0],
[0, 0, 1, 1, 0, 0,y1],
[0, 0, 1, 0, 1,y1, 0],
[0, 0, 1, 0, 1, 0,y1],
[1, 0, 0, 1, 0,y2, 0],
[1, 0, 0, 1, 0, 0,y2],
[1, 0, 0, 0, 1,y2, 0],
[1, 0, 0, 0, 1, 0,y2]]);
```